Optimising MBQC Patterns

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1 Background

The measurement-based quantum calculus [DKP07] is an alternative approach to the circuit-based formalism for describing quantum computation. It is less used but equivalent to it.

Computations in MBQC are sequences of two primitive operations, entanglement and measurement. This formalism is well -suited for describing analyses and optimisations of quantum computations, for verifiable blind quantum computation (VBQC, where the computation is delegated to a server without leaking information about what is actually computed) [BFK09, FK17, LMKO21], for benchmarking (some computations are universal and therefore at least as hard as any computations of a given size), and for targeting various kinds of hardware, such as trapped-ion and photonic platforms [GYC+11, BKB+12, LJZ+13].

We call an MBQC pattern an actual sequence of MBQC operations: qubit preparations, entanglements, and measurements. A given MBQC computation can be implemented by many patterns, which differ in their space (how many simultaneous living qubits they require), their depth (how long is a computation), and their lifetime (how long qubits have to remain alive). These are important trade-offs to consider in order to make the patterns runnable on actual hardware. Even if optimal results are known for some particular cases [HHF18], optimising MBQC patterns for space, for instance, is shown NP-hard via a reduction from interval thickness.

Interval thickness is a well-studied optimisation problem in compilation theory, especially for register allocation: a remarkable result is that, for a fixed maximum number of registers, finding a register allocation or proving that none exists can be performed in linear time [BGT98]. The problem of optimising MBQC patterns for space is slightly more general, however, and whether similar linear-time procedures exist remains open.

2 The Graphix Project

In the QAT team we are developing the Graphix library [GMU25], a platform for MBQC compilation, simulation and analysis. This platform supports a large part of our research activity. State-of-the art algorithms have been implemented for finding flows [MB24], and for optimal space minimisation in the case of a causal flow. Nevertheless, we are still looking for better heuristics in the general case.

3 The Internship Assignment

This internship focuses on discovering such heuristics and on investigating how results such as [BGT98] could be generalised to the context of MBQC patterns. The assignment involves combinatorial optimisation and some graph theory.

The team is looking for candidates with a strong interest in working at the boundaries between several research fields. Knowledge of quantum computing is not mandatory for this project.

References

[BFK09] Anne Broadbent, Joseph Fitzsimons, and Elham Kashefi. Universal Blind Quantum Computation. In *2009 50th Annual IEEE Symposium on Foundations of Computer Science*, pages 517–526, 2009.

[BGT98] Hans Bodlaender, Jens Gustedt, and Jan Arne Telle. Linear-time register allocation for a fixed number of registers. In *Proceedings of the Ninth Annual ACM-SIAM Symposium on Discrete Algorithms*, SODA '98, page 574–583, USA, 1998. Society for Industrial and Applied Mathematics.

[BKB+12] Stefanie Barz, Elham Kashefi, Anne Broadbent, Joseph F. Fitzsimons, Anton Zeilinger, and Philip Walther. Demonstration of Blind Quantum Computing. *Science*, 335(6066):303–308, Jan 2012.

[DKP07] Vincent Danos, Elham Kashefi, and Prakash Panangaden. The measurement calculus. *J. ACM*, 54(2):8–es, April 2007.

[FK17] Joseph F. Fitzsimons and Elham Kashefi. Unconditionally verifiable blind quantum computation. *Phys. Rev. A*, 96:012303, Jul 2017.

[GMU25] Maxime Garnier, Thierry Martinez, and Mateo Uldemolins. *Graphix: An Open-Source Compiler and Simulator for Measurement-Based Quantum Computation*, 2025.

[GYC+11] Wei-Bo Gao, Xing-Can Yao, Jian-Ming Cai, He Lu, Ping Xu, Tao Yang, Chao-Yang Lu, Yu-Ao Chen, Zeng-Bing Chen, and Jian-Wei Pan. Experimental measurement-based quantum computing beyond the cluster-state model. *Nature Photonics*, 5(2):117–123, Feb 2011.

[HHF18] Monireh Houshmand, Mahboobeh Houshmand, and Joseph F. Fitzsimons. Minimal qubit resources for the realization of measurement-based quantum computation. *Physical Review A*, 98(1), July 2018.

[LJZ+13] B. P. Lanyon, P. Jurcevic, M. Zwerger, C. Hempel, E. A. Martinez, W. Dür, H. J. Briegel, R. Blatt, and C. F. Roos. Measurement-Based Quantum Computation with Trapped Ions. *Phys. Rev. Lett.*, 111:210501, Nov 2013.

[LMKO21] Dominik Leichtle, Luka Music, Elham Kashefi, and Harold Ollivier. Verifying BQP Computations on Noisy Devices with Minimal Overhead. *PRX Quantum*, 2:040302, Oct 2021.

[MB24] Piotr Mitosek and Miriam Backens. An algebraic interpretation of pauli flow, leading to faster flow-finding algorithms, 2024.